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Non-BPS branes in a type-I orbifold

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Abstract: We analyse the spectrum of non-BPS branes in the type-I theory on the orbifold $\mathbb{T}^4/\mathcal{I}_4$. We present a detailed analysis of the action of the worldsheet parity Ω on the different D-brane boundary state sectors of type IIB on $\mathbb{T}^4/\mathcal{I}_4$, using the covariant formulation. Using these results we derive the spectrum of branes in the type-I orbifold. We find $\mathbb{Z}_2$- and $\mathbb{Z}$- charged non-BPS branes. A study of the stability of these branes in the type-I orbifold is also presented. We find that the type-I non-BPS D-particle and D-instanton remain stable in the orbifold. The D-particle carries no charge whereas the non-BPS D-instanton can carry twisted R-R charge.

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Contents

1. Introduction

In recent years, significant progress has been made towards understanding the various string duality symmetries. The validity of such dualities has often lead to new findings in string theory. For example, the strong-weak coupling duality between SO(32) heterotic string and the type-I theory has compelled to look for the states in type-I theory which are the dual partners of stable but non-BPS states in the heterotic string theory. A description of such states in type-I theory is found to be in terms of a tachyon kink solution on a D-brane anti-D-brane pair [1, 2, 3]. Subsequently, this phenomena and other duality relations have given rise to a tremendous activity in constructing and understanding the dynamics of non-BPS branes in string theories.[2]

[2]For reviews on non-BPS branes see [4]

The present understanding is that type-IIA (IIB) theory admits Dp-branes which are BPS for p even (odd) and non-BPS for p odd (even) for all values of p ranging from zero to nine. Furthermore, in both theories a descent relation has been established [5] involving BPS and non-BPS branes, i.e. a Dp-brane is seen as a descendant of a D$(p+1)$-brane. Moreover, one can obtain a Dp-brane of IIA (IIB) from a Dp-brane of IIB (IIA) by modding out with the discrete symmetry $(-1)^{F_L}$. BPS branes preserve half of the spacetime supersymmetries and are stable, whereas non-BPS branes break all the spacetime supersymmetries and are unstable. This instability can be explained by the presence of a tachyon in the open string spectrum of the brane. The effective action for these non-BPS branes, including the tachyon field, has been studied in [6]. Recently, a background independent effective action has been constructed for the case of non-commutative branes [7].

The unstable non-BPS branes of type-II theories can decay to the stable BPS branes via the condensation of a solitonic configuration of the tachyon field. This tachyon condensation has been extensively studied using string field theory [8] and p-adic string theory [9]. In the framework of non-commutative field theory [10] the above scenario is also understood via non-commutative solitons [11].

Although much of the above is understood in type-II theories, less is known for non-BPS branes in type-I theory, specially in orbifold backgrounds. The most interesting feature of the non-BPS branes in orbifold and orientifold theories is that they may be stable [1, 2, 3, 12, 13, 14]. This stability appears because of the fact that the tachyon state is projected out from the open string spectrum on the brane by the orbifold/orientifold symmetry.

In this paper, we study the spectrum of fractional and non-BPS branes in type-I theory on a $\mathbb{T}^4/\mathcal{I}_4$ orbifold. This orbifold is equivalent to the orientifold by the worldsheet parity-reversal Ω of type-IIB on $\mathbb{T}^4/\mathcal{I}_4$. Consistency of this type-I orbifold implies that the theory must contain D5- and D9-branes, with unitary and symplectic subgroups of $\mathrm{U}(16) \times \mathrm{U}(16)$ [15]–[19].

We follow the notation of [20] for D-branes where a Dp-brane in the orbifold $\mathbb{T}^4/\mathcal{I}_4$ of a type-II theory is denoted by $|D(r,s)\rangle$, $r+s=p$, where r denotes the number of spatial Neumann directions along the fixed plane and s denotes those along the orbifolded directions.[3]

We describe D-branes using the boundary state formalism, where D-branes are described by physical closed string states of the bosonic spectrum. In orbifold theories there will be twisted sectors as well and the D-branes may also contain boundary states in these sectors. Following [20], in type-II theories orbifolded by $\mathcal{I}_4$ (or $(-1)^{F_L} \cdot \mathcal{I}_4$) one finds BPS branes that can be either fractional or bulk branes. Fractional branes in type IIB on $\mathbb{T}^4/\mathcal{I}_4$ can exist for r odd and s even, and have boundary states

[3] For simplicity, for a $|D(r,s)\rangle$ and, unless otherwise stated, we will always choose the s Neumann directions starting from x^6; so that, for instance, a $D(1,2)$ is wrapped around the directions x^6 and x^7 of the $\mathbb{T}^4$.

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in all sectors of the theory:[4]

$$|D(r,s)\rangle_f = |D(r,s)\rangle_{\mathrm{NS,U}} + \epsilon_1 \, |D(r,s)\rangle_{\mathrm{R,U}} + \\ + \epsilon_2 \sum_{a=1}^{2^s} \mathrm{e}^{i\theta_a} \left(|D(r,s)\rangle_{\mathrm{NS,T_a}} + \epsilon_1 |D(r,s)\rangle_{\mathrm{R,T_a}} \right). \tag{1.1}$$

Each twisted sector is located in one of the 16 orbifold fixed planes. Furthermore, each boundary state sector is given by a GSO-invariant combination of boundary states.

Bulk branes can be described as branes with only untwisted sectors:

$$|D(r,s)\rangle_b = |D(r,s)\rangle_{\mathrm{NS,U}} + |D(r,s)\rangle_{\mathrm{R,U}} \,. \tag{1.2}$$

In general for type-II orbifold they exist for the same value as the fractional branes do, since two fractional branes with opposite twisted charges can join and give rise to a bulk brane. This brane can then move off the fixed planes.

Non-BPS branes in type-II $\mathbb{Z}_2$ orbifolds are referred to as truncated branes and are represented as follows:

$$|D(r,s)\rangle_t = |D(r,s)\rangle_{\mathrm{NS,U}} + \epsilon \sum_{a=1}^{2^s} \mathrm{e}^{i\theta_a} \, |D(r,s)\rangle_{\mathrm{R,T}} \,. \tag{1.3}$$

In type-IIB on $\mathbb{T}^4/\mathbb{Z}_2$ they exist for r and s odd [20]. They carry twisted R-R charge only, so they are referred to as $\mathbb{Z}$-charge non-BPS branes. In the conventions of this paper, these branes have a tension given by:

$$T_{(r,s)} = (\alpha')^{\frac{3-p}{2}} (2\pi)^{3-p} \sqrt{\pi} \,, \qquad r + s = p \,, \tag{1.4}$$

which coincides with the tension of a BPS Dp-brane in type IIA; and a (twisted R-R) charge

$$\tilde{Q}_{(r,s)} = (2\pi\sqrt{\alpha'})^{3-r} \pi^{-3/2} (\alpha')^{-1} \,, \tag{1.5}$$

which does not depend on the number of Neumann directions along the orbifolded 4-torus.

On the other hand, BPS branes in type-I theory are given in terms of boundary states as in type II with the addition of a 9-crosscap:[5]

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(|Dp\rangle_{\mathrm{NS}} + |Dp\rangle_{\mathrm{R}} + |C9\rangle \right). \tag{1.6}$$

[4]For short we use NS and R instead of NS-NS and R-R, respectively, as subindices in the boundary states.

[5]Here, and in subsequent formulae, the background D9 (and eventually the D5) branes are not explicitly displayed, but are understood.

The 9-crosscap is the closed string description of the orientifold 9-plane. BPS branes exist for $p = 1, 5, 9$, which are the values of p for which the corresponding R-R potential survives the Ω-projection.

Non-BPS branes in type I are given by just an NS-NS boundary state plus the crosscap contribution:

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(|Dp\rangle_{\rm NS} + |C9\rangle \right). \tag{1.7}$$

For $p = -1, 0, 7$ and 8 the contribution from the crosscap is such that the tachyons in the open strings ending on these non-BPS branes are projected out and thus giving the possibility of having stable non-BPS D-branes in type I [3, 13, 14].

The theory we consider in this article is type I on $\mathbb{T}^4/\mathcal{I}_4$, which can be seen either as an orbifold of type I or as an orientifold of the type-IIB orbifold. Therefore, the spectrum of D-branes of the type-I orbifold can be deduced by either applying the orbifold projection on the type-I D-branes or applying the Ω projection to the D-branes of the type-IIB orbifold. For the first case we must take into account as well the twisted sectors. Our approach consists in first studying the Ω projection on the boundary states of the type-IIB orbifold, for which we use the covariant formulation of the boundary states [21]. This analysis has not been performed before in the literature[6] and clarifies interesting subtleties about the action of Ω on the boundary states. Adding up the information about the $\mathcal{I}_4$ projection on the boundary states given in [20], we can deduce the boundary states which will survive in the type-I orbifold. The next step is to put together these states to make up the D-brane states of the theory. By the nature of the orbifold, we can deduce that there are bulk and fractional BPS D-branes. Regarding the non-BPS branes, some of them are truncated with twisted R-R charge, similar to the type-IIB orbifold. The rest of them have no R-R charge at all, like in the type-I theory before orbifolding. This latter type of brane can be seen as truncated branes of the type-IIB orbifold that get their R-R sector projected out by Ω, or else, as non-BPS branes of type I that did not get too much affected by the orbifold. On the other hand, those truncated branes of the type-I orbifold that have twisted R-R charge can be seen either as truncated branes of the type-IIB orbifold that are not modified by the Ω projection, or as non-BPS branes of type I that receive a contribution from the twisted sector.

The paper is organised as follows. In section 2 we present a thorough study of the action of Ω in the different boundary state sectors. A summary of the results can be found in table 1. Using these results, in section 3 we give a classification of the BPS and non-BPS branes of the type-I orbifold. This classification is summarised in tables 2 and 3. Section 4 is concerned about the stability of the non-BPS branes in the type-I orbifold. In section 5 we discuss the conclusions of our analysis. We include three appendices. Appendix A explains in detail the action of the operator

[6] The action Ω in the light-cone gauge has been studied for boundary states in [22], and for string states in [19].

Boundary State	Ω-invariance	$\mathcal{I}_4$-invariance
NS-NS untwisted	$\forall\ r,s$	$\forall\ r,s$
R-R untwisted	$r+s=1,5,9$	$s=$ even
R-R twisted	$r=-1,3$	$\forall\ r,s$
NS-NS twisted	$s=2$	$s=$ even

Table 1: Ω and $\mathcal{I}_4$-invariance of the boundary states. This table shows for which values of r and s each boundary state is invariant under the operations Ω and $\mathcal{I}_4$.

BPS branes	
Fractional & Bulk branes	$r=-1,3$ $s=2$
Bulk branes	$r=1,5$ $s=0,4$

Table 2: BPS branes in type I on $\mathbb{T}^4/\mathbb{Z}_2$. This table shows for which (r,s) we can have either BPS fractional or bulk branes in the type-I orbifold.

Non-BPS branes		
$\mathbb{Z}$-charge	$r=-1,3$ $s=0,1,3,4$	
$\mathbb{Z}_2$-charge	$r=1,5$ $s=0$	$r=$ even $s=0$

Table 3: Non-BPS branes in type I on $\mathbb{T}^4/\mathbb{Z}_2$. Non-BPS branes can have $\mathbb{Z}$-charge, if they have a twisted R-R sector, or $\mathbb{Z}_2$ charge if they do not have any R-R sector at all. The table shows for which (r,s) the boundary states exist in the type-I orbifold.

Ω on states in the asymmetric superghost picture $(-1/2,-3/2)$. Appendix B deals with the details of the action of Ω on the twisted NS-NS sector. Finally, appendix C contains details about the form of the crosscaps states used for the computations of this paper.

2. Action of Ω on the boundary states

We consider first the action of Ω on the open string sectors. From the mode expansion of the fields we have for the oscillators

$$\begin{aligned}\Omega\alpha_n^\mu\Omega^{-1} &= \pm e^{in\pi}\alpha_n^\mu\,,\\ \Omega\psi_m^\mu\Omega^{-1} &= \pm e^{im\pi}\psi_m^\mu\,,\end{aligned}\tag{2.1}$$

where the plus sign is for the NN directions and the minus sign is for the DD directions. Moreover, Ω relates the DN and ND strings so there is no definite action in these oscillators for these directions. Furthermore, for the NS-vacuum in the (-1) picture we have

$$\Omega|0\rangle_{-1} = -i|0\rangle_{-1}\,, \tag{2.2}$$

whereas the R-vacuum in the $(-1/2)$ picture transforms as:

$$\Omega|a\rangle_{-1/2} = -\Gamma^{\nu_{p+1}}\cdots\Gamma^{\nu_{9-p}}|a\rangle_{-1/2}\,, \tag{2.3}$$

where $\nu_{p+1},\ldots,\nu_{9-p}$ are the DD directions.

Regarding the oscillators of the closed string, for convenience, we define Ω as a combination of worldsheet parity-reversal ($\sigma \to 2\pi - \sigma$) and the GSO-projection. With this definition it has the same action on the physical states as worldsheet parity-reversal only. On the oscillators it acts by simply exchanging left and right sectors:

$$\Omega\alpha_n\Omega^{-1} = \tilde{\alpha_n}\,, \qquad \Omega\psi_n\Omega^{-1} = \tilde{\psi_n}\,, \tag{2.4}$$

and analogously for the right sector.

Similarly, Ω exchanges the left and right sectors of the (super)ghosts:

$$\begin{aligned}\Omega b_n\Omega^{-1} &= \tilde{b}_n\,, \qquad & \Omega c_n\Omega^{-1} &= \tilde{c}_n\,,\\ \Omega\beta_m\Omega^{-1} &= \tilde{\beta}_m\,, \qquad & \Omega\gamma_m\Omega^{-1} &= \tilde{\gamma}_m\,,\end{aligned} \tag{2.5}$$

and similarly for the right sector. The action of Ω on the NS-NS and R-R ground states for untwisted and twisted sectors are studied below, and the results are given in eqs. (2.7), (2.16), (2.34) and (A.19)

2.1 The untwisted NS-NS sector

The untwisted NS-NS boundary state[7]

$$|D(r,s),\eta\rangle_{\rm NS,U} = \frac{T_{(r,s)}}{2}\,|D(r,s)_X\rangle|D(r,s)_\psi,\eta\rangle_{\rm NS,U}|D(r,s)_{\rm gh}\rangle|D(r,s)_{\rm sgh},\eta\rangle_{\rm NS}\,, \tag{2.6}$$

is constructed upon the NS-NS ground state in the $(-1,-1)$ picture. The action of Ω on this ground state is given by:

$$\Omega\left(|0\rangle_{-1}\otimes\widetilde{|0\rangle}_{-1}\right) = \widetilde{|0\rangle}_{-1}\otimes|0\rangle_{-1} = -|0\rangle_{-1}\otimes\widetilde{|0\rangle}_{-1}\,. \tag{2.7}$$

On the other hand, since Ω exchanges left and right oscillators, the overall effect is a change from η to $-\eta$, hence we find:

$$\Omega|D(r,s),\eta\rangle_{\rm NS,U} = -|D(r,s),-\eta\rangle_{\rm NS,U}\,. \tag{2.8}$$

Therefore, the GSO projected untwisted NS-NS boundary state

$$|D(r,s)\rangle_{\rm NS,U} = \frac{1}{2}\left(|D(r,s),+\rangle_{\rm NS,U} - |D(r,s),-\rangle_{\rm NS,U}\right), \tag{2.9}$$

is invariant under Ω, for any $p = r+s$.

[7] We use the conventions of [23] for the definition of the boundary states. See also [24] for details.

2.2 The untwisted R-R sector

Consider now the untwisted R-R part. The R-R ground state in the $(-1/2,-1/2)$ picture has the following transformation property under Ω:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-1/2}\right)=\widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-1/2}=-|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-1/2}\,. \tag{2.10}$$

On the other hand, the R-R boundary state in the covariant formalism is most conveniently written in the asymmetric superghost picture $(-1/2,-3/2)$ [25]:

$$|D(r,s),\eta\rangle_{\mathrm{R,U}}=\frac{Q_{(r,s)}}{2}\,|D(r,s)_X\rangle|D(r,s)_\psi,\eta\rangle_{\mathrm{R,U}}|D(r,s)_{\mathrm{gh}}\rangle|D(r,s)_{\mathrm{sgh}},\eta\rangle_{\mathrm{R}}\,, \tag{2.11}$$

where $r+s=p$ odd in type IIB. The fermionic matter and the superghost components have zero-mode contributions

$$|D(r,s),\eta\rangle^{(0)}_{\mathrm{R,U}}=|D(r,s)_\psi,\eta\rangle^{(0)}_{\mathrm{R,U}}|D(r,s)_{\mathrm{sgh}},\eta\rangle^{(0)}_{\mathrm{R}}\,, \tag{2.12}$$

given by

$$\begin{aligned}|D(r,s),\eta\rangle^{(0)}_{\mathrm{R,U}}&=\mathrm{e}^{i\eta\gamma_0\tilde\beta_0}\mathcal{M}_{AB}\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,,\\ \mathcal{M}_{AB}(\eta)&=\left(\mathcal{C}_{(10)}\Gamma^0\cdots\Gamma^p\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB}\,,\qquad p=r+s\,,\end{aligned} \tag{2.13}$$

where A,B are spinor indices of $\mathrm{SO}(1,9)$ and $\mathcal{C}_{(10)}$ is the charge conjugation matrix in the corresponding representation of the 10-dimensional Γ-matrices.

Note that under the action of Ω, the superghost pictures seem to be swapped between left and right:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right)=\widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-3/2}=-|B\rangle_{-3/2}\otimes\widetilde{|A\rangle}_{-1/2}\,. \tag{2.14}$$

The commutation of the superghosts in order to recover the $(-1/2,-3/2)$ could yield a phase that we must calculate. In appendix A we give the details about how to obtain the action of Ω in the asymmetric picture. Here we present directly the result for the superghost sector:

$$\Omega|D(r,s)_{\mathrm{sgh}},\eta\rangle_{\mathrm{R}}=-i\eta|D(r,s)_{\mathrm{sgh}},-\eta\rangle_{\mathrm{R}}\,. \tag{2.15}$$

Regarding the fermion ground state we have

$$\Omega\left(|A\rangle\otimes\widetilde{|B\rangle}\right)=-|B\rangle\otimes\widetilde{|A\rangle}\,, \tag{2.16}$$

hence the effect of Ω on the matrix $\mathcal{M}_{AB}$ is a transposition:

$$\begin{aligned}\mathcal{M}(\eta)^T&=(-1)^{p+\frac12p(p+1)}\left(\mathcal{C}\Gamma^0\cdots\Gamma^p\frac{1+(-1)^pi\eta\Gamma_{11}}{1+i\eta}\right)\\&=-i\eta(-1)^{p+\frac12p(p+1)}\,\mathcal{M}(-\eta)\,.\end{aligned}$$

We then find

$$\Omega|D(r,s)_\psi,\eta\rangle^{(0)}_{\rm R} = -i\eta(-1)^{\frac{1}{2}p(p+1)}\,|D(r,s)_\psi,-\eta\rangle^{(0)}_{\rm R}\,. \tag{2.17}$$

In the oscillator part, Ω exchanges tilded oscillators with untilded ones, yielding an overall change of η to $-\eta$. Using this fact and the result (2.15) we finally find:

$$\begin{aligned}\Omega|D(r,s)_\psi,\eta\rangle_{\rm R}|D(r,s)_{\rm sgh},\eta\rangle_{\rm R} = -(-1)^{\frac{1}{2}p(p+1)}\,|D(r,s)_\psi,-\eta\rangle_{\rm R}\times\\ \times\,|D(r,s)_{\rm sgh},-\eta\rangle_{\rm R}\,,\end{aligned} \tag{2.18}$$

so that the GSO-invariant R-R boundary state

$$|D(r,s)\rangle_{\rm R,U} = \frac{1}{2}\left(|D(r,s),+\rangle_{\rm R,U} + |D(r,s),-\rangle_{\rm R,U}\right), \tag{2.19}$$

is Ω-invariant for $p=1,5,9$, as expected for the physical BPS D-branes in the type-I theory in the critical dimension.

Likewise, we can determine the Ω-projection on the D-branes by using the form of the quantum R-R string state in the $(-1/2,-3/2)$ picture,[8] given in terms of the R-R gauge potential:

$$\begin{aligned}|C^{(p+1)}\rangle = \frac{1}{2(p+1)!\sqrt{2}}\,C^{(p+1)}_{\mu_1\cdots\mu_{p+1}}\Big(\left(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_+\right)_{AB}\cos\gamma_0\tilde{\beta}_0\,+\\ +\left(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_-\right)_{AB}\sin\gamma_0\tilde{\beta}_0\Big)\times\\ \times|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,.\end{aligned} \tag{2.20}$$

We refer again to appendix A for the details of this derivation. We state here the result:

$$\Omega\,|C^{(p+1)}\rangle = -(-1)^{\frac{1}{2}p(p+1)}\,|C^{(p+1)}\rangle\,. \tag{2.21}$$

Thus it is invariant for $p=1,5,9$; the same values of p for which the R-R boundary states are Ω-invariant.

2.3 The twisted R-R sector

The orbifold of type IIB on $\mathbb{T}^4/\mathcal{I}_4$ has 16 6-dimensional fixed planes with $(2,0)$ supersymmetry each. The R-R twisted ground state consists of two spinors of SO$(1,5)$ of the same chirality, either $\mathbf{4}$ or $\mathbf{4}'$. Therefore, the twisted R-R sector can contain the following field strengths:

$$\mathbf{4}\otimes\mathbf{4} = [1]+[3]_+\,,\qquad \mathbf{4}'\otimes\mathbf{4}' = [1]+[3]_-\,. \tag{2.22}$$

Depending on which chirality we choose for the spinors, the 3-form field strength will be self-dual or anti self-dual. Accordingly, we have twisted R-R boundary states

[8] This particular approach presents the advantage over the usual one in the $(-1/2,-1/2)$ picture of making possible the description of spacetime-filling D-branes.

$|D(r,s)\rangle_{\rm R,T}$ for $r=-1,1,3$. As we will see below, only those states with $r=-1,3$ will survive the Ω projection, that is, only the twisted R-R scalar potential in (2.22) survives [18, 19]. However, note that (2.22) does not directly show the existence of a twisted 6-form potential, which was assumed in [19] and would be responsible for the twisted R-R tadpoles. This potential would be needed as well in order to account for an $r=5$ twisted R-R boundary state that one can construct, as we show below. We see by direct construction of the corresponding quantum state that such a R-R potential does exist but it does not survive the Ω projection.[9]

Consider first the action of Ω on the boundary states. The twisted R-R boundary state is given by

$$|D(r,s),\eta\rangle_{\rm R,T}=\frac{\tilde{Q}_{(r,s)}}{2}\,|D(r,s)_X\rangle_{\rm T}|D(r,s)_\psi,\eta\rangle_{\rm R,T}|D(r,s)_{\rm gh}\rangle|D(r,s)_{\rm sgh},\eta\rangle_{\rm R}\,,\quad (2.23)$$

which is constructed upon the twisted R-R ground state in the asymmetric $(-1/2,$ $-3/2)$ picture [24]. This sector contains zero-modes:

$$|D(r,s),\eta\rangle^{(0)}_{\rm R,T}={\rm e}^{i\eta\gamma_0\tilde{\beta}_0}M_{ab}\,|a\rangle_{-1/2,{\rm T}}\otimes\widetilde{|b\rangle}_{-3/2,{\rm T}}\,,\qquad (2.24)$$

with

$$M_{ab}(\eta)=\left(\mathcal{C}_{(6)}\gamma^0\cdots\gamma^r\frac{1+i\eta\gamma}{1+i\eta}\right)_{ab}\,,\qquad (2.25)$$

where a,b are spinor indices of ${\rm SO}(1,6)$ and $\mathcal{C}_{(6)}$ is the charge conjugation matrix associated to the ${\rm SO}(1,5)$ γ-matrices, for which we use the same conventions as in [24].

The action of Ω on this ground state results as before an exchange of pictures between left and right, hence we could proceed as before. Since the superghost part is the same as in the untwisted sector, the action of Ω on the $(-1/2,-3/2)$ picture is the same as given before in (2.15), and the effect on the zero-mode matrix M_{ab} is again a transposition:

$$\begin{aligned}M(\eta)^T&=-(-1)^{r+\frac{1}{2}r(r+1)}\left(\mathcal{C}_{(6)}\gamma^0\cdots\gamma^r\frac{1+(-1)^ri\eta\gamma}{1+i\eta}\right)\\&=-i\eta(-1)^{\frac{1}{2}r(r+1)}\,M(-\eta)\,,\end{aligned}$$

where we have used the fact that fractional and truncated branes in type IIB on $\mathbb{T}^4/\mathcal{I}_4$ only exist for r odd. Finally, using the above results and the fact that the twisted spinor indices anti-commute [26, 27], we find:

$$\Omega|D(r,s),\eta\rangle^{(0)}_{\rm R,T}=(-1)^{\frac{1}{2}r(r+1)}\,|D(r,s),-\eta\rangle^{(0)}_{\rm R,T}\,,\qquad (2.26)$$

[9]On the other hand, one could choose the projection $\Omega=-1$ in the twisted sector, defined as the $\Omega\mathcal{J}$ projection in [26]. In that case the twisted R-R 6-form potential would survive in the orientifold.

hence the GSO-invariant twisted R-R boundary state

$$|D(r,s)\rangle_{\mathrm{R,T}} = \frac{1}{2}\left(|D(r,s),+\rangle_{\mathrm{R,T}} + |D(r,s),-\rangle_{\mathrm{R,T}}\right), \tag{2.27}$$

is Ω invariant for $r=-1,3$. For fractional branes s is even and for truncated branes s is odd [20]. Thus we find that there are non-BPS (truncated) branes $|D(-1,s)\rangle$ and $|D(3,s)\rangle$ branes, for s odd, in type I on $\mathbb{T}^4/\mathcal{I}_4$. For the fractional (BPS) branes we must still find out which NS-NS twisted boundary states survive the projection. This is done in the next subsection.

We can find the same result using the quantum states of the R-R twisted sector in the $(-1/2,-3/2)$ picture:

$$\begin{aligned}|C^{(r+1)}\rangle_{\mathrm{T}} = \frac{1}{\sqrt{2}(p+1)!} C^{(r+1)}_{\mu_1\cdots\mu_{r+1}} \Big(\left(\mathcal{C}_{(6)}\gamma^{\mu_1\cdots\mu_{p+1}}\Pi_+\right)_{ab}\cos\gamma_0\tilde{\beta}_0 + \\ + \left(\mathcal{C}_{(6)}\gamma^{\mu_1\cdots\mu_{p+1}}\Pi_-\right)_{ab}\sin\gamma_0\tilde{\beta}_0\Big)\times \\ \times |a\rangle_{-1/2,\mathrm{T}} \otimes \widetilde{|b\rangle}_{-3/2,\mathrm{T}}\,. \end{aligned} \tag{2.28}$$

Using (A.20) from appendix A we find

$$\Omega\,|C^{(r+1)}\rangle_{\mathrm{T}} = (-1)^{\frac{1}{2}r(r+1)}\,|C^{(r+1)}\rangle_{\mathrm{T}}, \tag{2.29}$$

hence the twisted scalar and its dual, the 4 form potential, survive the Ω projection, whereas the 6-form potential does not survive, which is in agreement with the results of [17, 18, 19].

2.4 The twisted NS-NS sector

The boundary state in the twisted NS-NS sector

$$\begin{aligned}|D(r,s),\eta\rangle_{\mathrm{NS,T}} = \frac{\tilde{T}_{(r,s)}}{2}\,|D(r,s)_X\rangle_{\mathrm{T}}|D(r,s)_\psi,\eta\rangle_{\mathrm{NS,T}}|D(r,s)_{\mathrm{gh}}\rangle \times \\ \times |D(r,s)_{\mathrm{sgh}},\eta\rangle_{\mathrm{NS}}\end{aligned} \tag{2.30}$$

is constructed upon the NS-NS ground state in the $(-1,-1)$ superghost picture. This boundary state contains a sector for the fermion zero-modes coming from the compact directions 6, 7, 8 and 9:

$$|D(r,s),\eta\rangle^{(0)}_{\mathrm{NS,T}} = m_{\alpha\beta}(\eta)\,|\alpha\rangle_{-1,\mathrm{T}} \otimes \widetilde{|\beta\rangle}_{-1,\mathrm{T}}, \tag{2.31}$$

where α,β are spinor indices of SO(4). Furthermore, from the overlapping condition for the zero modes, we find:

$$m_{\alpha\beta}(\eta) = \left(\mathcal{C}_{(4)}\Pi_{(s)}\frac{1-i\eta\bar{\gamma}}{1-i\eta}\right)_{\alpha\beta}, \tag{2.32}$$

where $\Pi_{(s)}$ is the product of s gamma-matrices:

$$\Pi_{(s)} = \bar{\gamma}^6 \bar{\gamma}^7 \cdots \cdot , \qquad s \neq 0 ,$$
$$\Pi_{(s)} = 1 , \qquad s = 0 , \tag{2.33}$$

and $\bar{\gamma} = -\bar{\gamma}^6\bar{\gamma}^7\bar{\gamma}^8\bar{\gamma}^9$ is the chirality matrix in four euclidean dimensions. Moreover, $\mathcal{C}_{(4)}$ represents the charge-conjugation matrix in the present context. Our conventions for the SO(4) gamma-matrices and the action of the fermionic zero-modes on the ground state can be found in appendix B.

The ground state, which is constructed with anti-commuting spin fields of the same chirality, transforms as follows under Ω:

$$\Omega \left(|\alpha\rangle_{-1,\mathrm{T}} \otimes \widetilde{|\beta\rangle}_{-1,\mathrm{T}} \right) = (\bar{\gamma})^{\alpha}{}_{\delta} \, |\beta\rangle_{-1,\mathrm{T}} \otimes \widetilde{|\delta\rangle}_{-1,\mathrm{T}} = (\bar{\gamma})^{\beta}{}_{\delta} \, |\delta\rangle_{-1,\mathrm{T}} \otimes \widetilde{|\alpha\rangle}_{-1,\mathrm{T}} \, . \tag{2.34}$$

A proof of this result is presented in appendix B. Using this result we obtain the following action of Ω on the boundary state corresponding to the vacuum sector:

$$\begin{aligned} \Omega \, |D(r,s),\eta\rangle^{(0)}_{\mathrm{NS,T}} &= \left(m(\eta)^T \, \bar{\gamma} \right)_{\alpha\beta} |\alpha\rangle_{-1,\mathrm{T}} \otimes \widetilde{|\beta\rangle}_{-1,\mathrm{T}} \\ &= -(-1)^{\frac{1}{2}s(s-1)} \, |D(r,s),-\eta\rangle^{(0)}_{\mathrm{NS,T}} \, . \end{aligned} \tag{2.35}$$

with s even. Note that the twisted NS-NS boundary state is $\mathcal{I}_4$-invariant for $s =$ even [20]. On the other hand, in the sector bilinear in the oscillators Ω acts again by changing η by $-\eta$. Thus we finally deduce:

$$\Omega \, |D(r,s),\eta\rangle_{\mathrm{NS,T}} = -(-1)^{\frac{1}{2}s(s-1)} \, |D(r,s),-\eta\rangle_{\mathrm{NS,T}} \, , \tag{2.36}$$

hence the GSO-projected boundary state

$$|D(r,s)\rangle_{\mathrm{NS,T}} = \frac{1}{2} \left(|D(r,s),+\rangle_{\mathrm{NS,T}} + |D(r,s),-\rangle_{\mathrm{NS,T}} \right) , \qquad s = \text{even} \tag{2.37}$$

is Ω-invariant only for $s = 2$. Accordingly, only branes with $s = 2$ in type I on $\mathbb{T}^4/\mathcal{I}_4$ carry a NS-NS twisted sector.

3. Non-BPS branes in type I on $\mathbb{T}^4/\mathbb{Z}_2$

Before describing in detail the stability of the non-BPS branes in the type-I orbifold, we summarise the brane spectrum that we have found with the analysis based on the boundary states. Recall that the D-branes are denoted by $|D(r,s)\rangle$, with r the number of Neumann directions along the orbifold fixed plane and s the number of Neumann directions along $\mathbb{T}^4$. From the point of view of the closed string the orientifold is described in terms of crosscaps. Since there are orientifold 9- and 5-planes, we must consider both 9-crosscap $|C9\rangle$ and 5-crosscaps $|C5\rangle$. The type-I orbifold can be considered as an orientifold of type IIB on $\mathbb{T}^4/\mathcal{I}_4$. Accordingly the boundary states for the branes in the type-I orbifold theory will be modified by the addition of the crosscaps:

$$|D\rangle \longrightarrow |D\rangle_{\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D\rangle + |C9\rangle + |C5\rangle\right), \tag{3.1}$$

where we would introduce one 5-crosscap for each fixed plane, and we have chosen this particular normalisation to account for the normalisation of the orientifold projectors in the open string channel.

In type I on $\mathbb{T}^4/\mathbb{Z}_2$ we find also BPS fractional branes, which have states in all sectors twisted and untwisted, and include the crosscaps as well:

$$|D(r,s)\rangle_{f,\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_f + |C9\rangle + |C5\rangle\right). \tag{3.2}$$

The values of r and s for which they exist are such that the boundary state in every sector is invariant under both Ω and $\mathcal{I}_4$ besides being GSO-invariant. From the results of the previous section, shown in table 1, we find these values to be $r = -1, 3$ and $s = 2$, i.e. the orbifold theory has an *instantonic* D1 and a D5 as fractional branes.

BPS Bulk branes can be described as branes with only untwisted sectors. A possibility is to have r and s such that the untwisted sectors are invariant, but not the twisted sectors. This yields $r = 1, 5$ and $s = 0, 4$, which accounts for D1-, D5- and D9-branes. On the other hand, two fractional branes with opposite twisted R-R charge can join and produce a bulk brane that can move off the fixed planes. Therefore we also can have bulk branes for the same values of r and s as for fractional branes. Bulk branes can be represented as follows:

$$|D(r,s)\rangle_{b,\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_b + |C9\rangle + |C5\rangle\right). \tag{3.3}$$

In the type-I orbifold, only those truncated branes whose twisted R-R state survives the orbifold symmetry are $\mathbb{Z}$-charged non-BPS branes. Moreover, some non-BPS branes of type I have values r and s such that the twisted R-R charge in the type-I orbifold exist. Thus these branes are also truncated in the orbifold. We find that $\mathbb{Z}$-charged non-BPS branes can exist for $r = -1, 3$ and $s = 0, 1, 3, 4$, since for these values $|D(r,s)\rangle_{\mathrm{R,T}}$ is Ω-invariant but $|D(r,s)\rangle_{\mathrm{R}}$ is not. These branes can be represented by:

$$|D(r,s)\rangle_{t,\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_t + |C9\rangle + |C5\rangle\right). \tag{3.4}$$

These branes include the non-BPS D-instanton and D7-brane of type I, which have twisted R-R charge in the type-I orbifold. This $\mathbb{Z}$-charged D7-brane appears only for the orientation $(3, 4)$.

The last possibility consists of those branes which have neither untwisted nor twisted R-R charge. These include those branes which are BPS in the type-IIB orbifold, but become non-BPS in the orientifold, and also those branes in type I which do not obtain any R-R charge after the orbifolding. They are described solely by an untwisted NS-NS sector, and in the type-I orbifold can be represented by:

$$|D(r,s)\rangle_{t,\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_{\mathrm{NS,U}} + |C9\rangle + |C5\rangle\right). \tag{3.5}$$

In principle, the values of r and s for which they can exist are those for which no R-R sector survives. The two possibilities would be $r = 1, 5$ and $s = 1, 2, 3$, or $r =$even (including $r = 0$) and $s = 0, \ldots, 4$. These branes would include the non-BPS D0, as well the D7- and D8-branes of type I, which can appear with different orientations, namely, $(5, 2)$, $(4, 3)$, $(5, 3)$, $(4, 4)$. However, as will be explained in the next section, only $(r, 0)$ branes can be properly defined as $\mathbb{Z}_2$-charged branes. We will also prove in the next section that only the $(0, 0)$ and the $(-1, 0)$ branes are in fact stabilised by the action of the orientifold.

4. Stability of the non-BPS branes

In this section we study the stability of the non-BPS branes presented in the previous section. For this analysis it is crucial to notice that the tadpole-cancelling D9- and D5-branes in type I on $\mathbb{T}^4/\mathcal{I}_4$ are bulk branes[10] The normalisation of the boundary states in the orbifold involves the trace of the representation matrices of the orientifold group on the open string sector. This trace is exactly zero for the case of the twisted R-R 6-form, hence the D9- and D5-branes do not carry a twisted R-R sector, and by supersymmetry they do not carry any twisted NS-NS sector either. Therefore the 9- and 5- crosscaps of the type-I orbifold have boundary states in the untwisted sectors only:[11]

$$|C9\rangle = |C9\rangle_{\mathrm{NS,U}} + |C9\rangle_{\mathrm{R,U}}\,, \qquad |C5\rangle = |C5\rangle_{\mathrm{NS,U}} + |C5\rangle_{\mathrm{R,U}}\,. \tag{4.1}$$

The detailed form of these crosscaps is given in appendix C. We start by carrying out first in detail the analysis of the stability for the $\mathbb{Z}_2$-charged D-particle, which will illustrate the more general case for (r, s) branes considered next. We then finish by considering the $\mathbb{Z}$-charged branes.

4.1 The stable non-BPS D-particle with $\mathbb{Z}_2$ charge

In this subsection we explicitly carry out the computations for the D-particle and establish the formalism to show various consistencies. In the next subsection, this analysis can be simply used to obtain the required results for (r, s) branes without

[10]This can be deduced from the fact that the twisted R-R state associated with the 6-form potential is odd under Ω, which agrees with the fact that the tadpole associated with the twisted 6-form is identically zero [17, 18, 19].

[11]Consistency conditions of type I on the orbifold $\mathbb{T}^4/\mathcal{I}_4$ using the boundary state formalism was first considered in [16].

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repeating the technical details. The boundary state for the D-particle in the orbifold theory is given by:

$$|D0\rangle_{\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D0\rangle_{\mathrm{NS,U}} + |C9\rangle + |C5\rangle\right), \tag{4.2}$$

where $|D0\rangle_{\mathrm{NS,U}}$ denotes the GSO and $\mathcal{I}_4$ invariant NS-NS boundary state written as a linear combination of the D-particle and its image in type IIB on $\mathbb{T}^4/\mathcal{I}_4$. Thus we have

$$\begin{aligned}|D0\rangle_{\mathrm{NS,U}} = \frac{1}{2\sqrt{2}}\Big(&|D0, a^\alpha, y^i, +\rangle_{\mathrm{NS,U}} + |D0, a^\alpha, -y^i, +\rangle_{\mathrm{NS,U}} - \\ &- |D0, a^\alpha, y^i, -\rangle_{\mathrm{NS,U}} - |D0, a^\alpha, -y^i, -\rangle_{\mathrm{NS,U}}\Big),\end{aligned} \tag{4.3}$$

with a^α and y^i the positions of the D-particle in the non compact and compact directions, respectively. For these branes, it is understood that y^i is not the origin. The crosscap states are similarly taken in GSO invariant combinations.

We want to test whether there are any tachyons in the open string spectrum. The relevant open string amplitudes to be considered are given by the annulus and Möbius amplitudes, which can be obtained from the following closed string amplitudes:

$$\begin{aligned}\mathcal{A} &= \frac{1}{2}{}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|D0\rangle_{\mathrm{NS,U}}\,, \\ \mathcal{M}_9 + \mathcal{M}_5 &= \frac{1}{2}{}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|C9\rangle + \frac{1}{2}{}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|C5\rangle + c.c\,,\end{aligned} \tag{4.4}$$

where $\mathcal{D}$ is the closed string propagator

$$\mathcal{D} = \frac{\alpha'}{4\pi}\int_{|z|\le 1}\frac{d^2z}{|z|^2}\, z^{L_0-a}\bar{z}^{\tilde{L}_0-a}\,, \tag{4.5}$$

where $a = 1/2$ for the untwisted NS-NS sector and $a = 0$ in the R-R sector (twisted and untwisted) and the twisted NS-NS sector. The crosscaps will interact with the $\mathbb{Z}_2$-charged D-particle only through the untwisted NS-NS sector.

Note that the normalisation of the brane is modified by the fact that the orbifold is compact, namely:

$$|D0, \eta\rangle_{\mathrm{NS,U}} = N_0\,|D0_X\rangle|D0_\psi, \eta\rangle_{\mathrm{NS,U}}|D0_{\mathrm{gh}}\rangle|D0_{\mathrm{sgh}}, \eta\rangle_{\mathrm{NS}}\,, \tag{4.6}$$

with

$$N_0 = \sqrt{2}\frac{T_0}{2}\left(\frac{2\pi R}{\Phi}\right)^2 (2\pi R)^{-4}\,, \tag{4.7}$$

where the radius of each of the four compact coordinates is taken to be R and T_0 is the tension of the BPS D-particle in type-IIA theory:

$$T_0 = \sqrt{\pi}(2\pi\sqrt{\alpha'})^3\,. \tag{4.8}$$

We have introduced the self-dual volume Φ defined as[12]

$$\langle n, m | n' m' \rangle = \Phi \, \delta_{nn'} \delta_{mm'} \, , \tag{4.9}$$

for each compact direction. Moreover, the bosonic sector is modified as follows:

$$|D0_X\rangle = \delta^{(5)}(q^\alpha - a^\alpha) \left(\sum_{n \in \mathbb{Z}} \mathrm{e}^{iq_n \frac{n}{R}} \right)^4 \times \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n} \alpha_{-n} \cdot \mathcal{S} \cdot \tilde{\alpha}_{-n}} \, |k, m, n = 0\rangle \, . \tag{4.10}$$

With this information and using the explicit form of the crosscaps given in appendix C, we can compute the required amplitudes. After standard operations we find in the closed string channel

$$\begin{aligned} {}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|D0\rangle_{\mathrm{NS,U}} = & \frac{\alpha'\pi}{2} \frac{N_0^2}{2} V_1 \Phi^4 (2\pi^2\alpha')^{-5/2} \int_0^\infty d\ell \, \ell^{-5/2} \times \\ & \times \left\{ \left(\sum_{n \in \mathbb{Z}} \mathrm{e}^{-\pi \ell \frac{\alpha'}{2} (\frac{n}{R})^2} \right)^4 + \left(\sum_{n \in \mathbb{Z}} \mathrm{e}^{-\pi \ell \frac{\alpha'}{2} (\frac{n}{R})^2 - 2i \frac{yn}{R}} \right)^4 \right\} \times \\ & \times \left\{ \left(\frac{f_3(q)}{f_1(q)} \right)^8 - \left(\frac{f_4(q)}{f_1(q)} \right)^8 \right\} , \end{aligned} \tag{4.11}$$

with $q = \mathrm{e}^{-\pi\ell}$. The closed string amplitudes related to the Möbius strips are given by:

$$\begin{aligned} {}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|C9\rangle_{\mathrm{NS,U}} = & -\frac{\alpha'\pi}{2} \frac{T_0}{4\sqrt{2}} \frac{T_9}{2} V_1 \, 2^5 2^{9/2} \int_0^\infty d\ell \, \times \\ & \times \left\{ \left(\frac{f_1(iq)}{f_3(iq)} \right) \left(\frac{f_4(iq)}{f_2(iq)} \right)^9 - \left(\frac{f_1(iq)}{f_4(iq)} \right) \left(\frac{f_3(iq)}{f_2(iq)} \right)^9 \right\} \end{aligned} \tag{4.12}$$

and

$$\begin{aligned} {}_{\mathrm{NS,U}}\langle D0|\mathcal{D}|C5\rangle_{\mathrm{NS,U}} = & \frac{\alpha'\pi}{4} \frac{T_0}{4\sqrt{2}} \frac{T_5}{2} (2\pi R)^{-4} \, V_1 \, 2^{5/2} \int_0^\infty d\ell \, \times \\ & \times \left\{ \left(\sum_{n \in \mathbb{Z}} \mathrm{e}^{-\pi \ell \frac{\alpha'}{2} (\frac{n}{R})^2 - i \frac{yn}{R}} \right)^4 + \left(\sum_{n \in \mathbb{Z}} \mathrm{e}^{-\pi \ell \frac{\alpha'}{2} (\frac{n}{R})^2 + i \frac{yn}{R}} \right)^4 \right\} \times \\ & \times \left\{ \left(\frac{f_3(iq)}{f_1(iq)} \right)^3 \left(\frac{f_4(iq)}{f_2(iq)} \right)^5 - \left(\frac{f_4(iq)}{f_1(iq)} \right)^3 \left(\frac{f_3(iq)}{f_2(iq)} \right)^5 \right\} , \end{aligned} \tag{4.13}$$

In order to obtain the amplitudes in open string channel we perform a modular transformation, $\ell = t^{-1}$, for the cylinder and $\ell = (4t)^{-1}$ for the Möbius strip. After

[12] See [23] for more details.

using the modular properties of the functions[13] f_i, we find:

$$\begin{aligned}
\mathcal{A} = V_1 (8\pi^2\alpha')^{-1/2} \int_0^\infty \frac{dt}{2t} t^{-1/2} \; \times \\
\times \left\{ \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-\frac{2\pi t}{\alpha'}(mR)^2} \right)^4 + \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-\frac{2\pi t}{\alpha'}R^2(m+\frac{y}{\pi R})^2} \right)^4 \right\} \times \\
\times \left\{ \left(\frac{f_3(\tilde q)}{f_1(\tilde q)} \right)^8 - \left(\frac{f_2(\tilde q)}{f_1(\tilde q)} \right)^8 \right\},
\end{aligned} \tag{4.14}$$

with $\tilde q = \mathrm{e}^{-\pi t}$.

$$\begin{aligned}
\mathcal{M}_9 = V_1\, 2^4\, (8\pi^2\alpha')^{-1/2} \int_0^\infty \frac{dt}{2t} t^{-1/2} \; \times \\
\times \left\{ \mathrm{e}^{-i\frac{\pi}{4}} \left(\frac{f_1(i\tilde q)}{f_4(i\tilde q)} \right) \left(\frac{f_3(i\tilde q)}{f_2(i\tilde q)} \right)^9 - \mathrm{e}^{i\frac{\pi}{4}} \left(\frac{f_1(i\tilde q)}{f_3(i\tilde q)} \right) \left(\frac{f_4(i\tilde q)}{f_2(i\tilde q)} \right)^9 \right\}
\end{aligned} \tag{4.15}$$

and

$$\begin{aligned}
\mathcal{M}_5 = V_1\, 2^2\, (8\pi^2\alpha')^{-1/2} \int_0^\infty \frac{dt}{2t} t^{-1/2} \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-\frac{8\pi t}{\alpha'}R^2(m+\frac{y}{2\pi R})^2} \right)^4 \times \\
\times \left\{ \mathrm{e}^{-i\frac{\pi}{4}} \left(\frac{f_4(i\tilde q)}{f_1(i\tilde q)} \right)^3 \left(\frac{f_3(i\tilde q)}{f_2(i\tilde q)} \right)^5 - \mathrm{e}^{i\frac{\pi}{4}} \left(\frac{f_3(i\tilde q)}{f_1(i\tilde q)} \right)^3 \left(\frac{f_4(i\tilde q)}{f_2(i\tilde q)} \right)^5 \right\}.
\end{aligned} \tag{4.16}$$

The open string spectrum for these non-BPS branes in the type-I orbifold is given by the total amplitude $\mathcal{A}_{\text{total}} = \mathcal{A} + \mathcal{M}_9 + \mathcal{M}_5$. These amplitudes can be checked to correspond to the following amplitudes in the open string channel:

$$\begin{aligned}
\mathcal{A} &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS},R} \left(\frac{1}{4} (-1)^{F_s} \mathrm{e}^{-2\pi t L_0} \right), \\
\mathcal{M}_9 &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS}} \left(\frac{1}{4} \Omega\, \mathrm{e}^{-2\pi t L_0} \right), \\
\mathcal{M}_5 &= \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS}} \left(\frac{1}{4} \mathcal{I}_4 \Omega\, \mathrm{e}^{-2\pi t L_0} \right),
\end{aligned} \tag{4.17}$$

where besides the trace over the oscillators, the Tr in each case includes the trace of Chan-Paton factors, integration over momenta in non-compact directions and summing over discrete momenta and winding states. The total amplitude can therefore be written in the following compact form:

$$\mathcal{A}_{\text{total}} = \int_0^\infty \frac{dt}{2t} \mathrm{Tr}_{\mathrm{NS},R} \left\{ (-1)^{F_s} \left(\frac{1+\mathcal{I}_4}{2} \right) \left(\frac{1+\Omega}{2} \right) \mathrm{e}^{-2\pi t L_0} \right\}. \tag{4.18}$$

[13]The explicit form of the modular properties of these functions with imaginary argument can be found in [14].

In this expression one must take into account that $\mathcal{I}_4$ in the NS sector yields zero and in the R sector $\mathcal{I}_4$, Ω and $\mathcal{I}_4\Omega$ are zero. Note that it is crucial to take the linear combination of the brane and its image as the boundary state to get the non-trivial vanishing result for the $\mathcal{I}_4$ projection while still getting a non-vanishing contribution for $\mathcal{I}_4\Omega$. This is essential for the closed-open consistency condition for the orbifold theory. This also reflects that we need to include the C-5 state to account for the interaction between the brane and the O5 planes.

We can now check that the tachyon state cancels out from the open string spectrum by taking the limit $t \to \infty$. In this limit we obtain the following leading terms in the amplitude:

$$\begin{aligned} \mathcal{A} &\simeq V_1(8\pi^2\alpha')^{-1/2}\int\frac{dt}{2t}t^{-1/2}\,\tilde{q}^{-1}\left(1+\mathrm{e}^{-\frac{8t}{\pi\alpha'}y^2}\right), \\ \mathcal{M}_9 &\simeq -V_1(8\pi^2\alpha')^{-1/2}\int\frac{dt}{2t}t^{-1/2}\,\tilde{q}^{-1}, \\ \mathcal{M}_5 &\simeq -V_1(8\pi^2\alpha')^{-1/2}\int\frac{dt}{2t}t^{-1/2}\,\tilde{q}^{-1}\left(\mathrm{e}^{-\frac{8t}{\pi\alpha'}y^2}\right), \end{aligned} \tag{4.19}$$

hence $\mathcal{A}_{\mathrm{total}} \simeq 0$, and the tachyon cancels. It is important to note that the contribution from the D-particle-C5 amplitude cancels with the winding (y-dependent) part in the particle-image-particle amplitude and the contribution from the particle-C9 amplitude cancels with the y-independent part. The reason for such a mechanism is that the O9 plane, being space-time filling, is not sensitive to the position of the brane. On the contrary, the C5-state is localised at the origin, hence it is sensitive to the position of the brane. This mechanism will be relevant in the next subsection to find other stable non-BPS $\mathbb{Z}_2$-charged branes.

Note that there are also open strings that stretch between the D-particle and the background D5- and D9-branes. However, these do not have any tachyon because the intercept in the NS-sector for these strings is strictly negative since

$$a_{\mathrm{NS}} = \frac{1}{2} - \frac{ND}{2}, \tag{4.20}$$

where ND is the number of mixed Dirichlet-Neumann directions and is 9 and 5 for D0-D9 and D0-D5 strings, respectively. Thus the D-particle in this orbifold theory is absolutely stable as it is in the case before orbifolding.

4.2 Stable non-BPS branes with $\mathbb{Z}_2$ charge

We now generalise the previous results for a generic $\mathbb{Z}_2$ brane. We note that (r, s) branes with s different from zero cannot be properly defined as $\mathbb{Z}_2$-charged branes. The reason is that for non-zero s we are able to distinguish a brane and its image only by introducing a Wilson line. This is consistent with T-duality but requires, however, the brane to have a twisted sector,[14] i.e. the brane must be either a fractional or $\mathbb{Z}$-charged. Thus only $(r,0)$ branes can be formulated as $\mathbb{Z}_2$ charged branes. The open-closed duality consistency works for these branes just as the case for the D-particle and we will not repeated again. The stability analysis follows closely to the procedure used in [14] for the uncompactified theory. We introduce a parameter m_r to renormalise the tension of these branes, which will be fixed by imposing that there are no tachyons in the open string spectrum on that brane which is also consistent with open-closed consistency as we will see below. In order to achieve this, we calculate the interaction between two of these non-BPS branes and translate the result to open string channel. Then, we impose that there are no tachyons in the open string spectrum, which will fix the parameter m_r. Only positive values of m_r will be considered to produce a consistent stable brane.

In the orientifold, the $\mathbb{Z}_2$-charged branes are thus represented as follows:

$$|D(r,0)\rangle_{\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,0)\rangle_{\mathrm{NS,U}} + |C9\rangle + |C5\rangle\right). \tag{4.21}$$

where, as before, the state $|D(r,0)\rangle_{\mathrm{NS,U}}$ represents the the NS-NS sector of the brane in type IIB on $\mathbb{T}^4/\mathcal{I}_4$, hence

$$\begin{aligned}|D(r,0)\rangle_{\mathrm{NS,U}} = \frac{1}{2\sqrt{2}}\Big(&|D(r,0),a^\alpha,y^i,+\rangle_{\mathrm{NS,U}} + |D(r,0),a^\alpha,-y^i,+\rangle_{\mathrm{NS,U}} - \\ &- |D(r,0),a^\alpha,y^i,-\rangle_{\mathrm{NS,U}} - |D(r,0),a^\alpha,-y^i,-\rangle_{\mathrm{NS,U}}\Big), \end{aligned} \tag{4.22}$$

with a^α being the position of the brane in the $(5-r)$ non-compact transverse directions and y^i is that of the 4 compact transverse directions. The relevant open string amplitudes are given by the annulus and Möbius amplitudes, which can be obtained from the closed channel as before:

$$\begin{aligned}\mathcal{A} &= \frac{1}{2}{}_{\mathrm{NS,U}}\langle D(r,0)|\mathcal{D}|D(r,0)\rangle_{\mathrm{NS,U}}\,,\\ \mathcal{M}_9 + \mathcal{M}_5 &= \frac{1}{2}{}_{\mathrm{NS,U}}\langle D(r,0)|\mathcal{D}|C9\rangle + \frac{1}{2}{}_{\mathrm{NS}}\langle D(r,0)|\mathcal{D}|C5\rangle + \mathrm{c.c.}\,.\end{aligned} \tag{4.23}$$

We normalise the brane in the following way:

$$|D(r,0),\eta\rangle_{\mathrm{NS,U}} = N_p\,|D(r,0)_X\rangle|D(r,0)_\psi,\eta\rangle_{\mathrm{NS,U}}|D(r,0)_{\mathrm{gh}}\rangle|D(r,0)_{\mathrm{sgh}}\rangle_{\mathrm{NS}}\,, \tag{4.24}$$

with

$$N_r = m_r\frac{T_r}{2}\left(\frac{2\pi R}{\Phi}\right)^2(2\pi R)^{-4}\,, \tag{4.25}$$

[14]The Wilson line creates a flux on the world volume. In the presence of the orbifold, since the string closes only up to a phase, there is a flux deficit. In order to compensate for this a twisted sector must be then introduced.

where T_r is the tension of a BPS Dr-brane in type-II theory:

$$T_r = \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-r}\,. \tag{4.26}$$

Moreover, the bosonic sector is modified as follows:

$$\begin{aligned} |D(r,0)_X\rangle = {} & \delta^{(5-r)}(q^\alpha - a^\alpha)\left(\sum_{n\in\mathbb{Z}} \mathrm{e}^{iq_n n/R}\right)^4 \times \\ & \times \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n}\alpha_{-n}\cdot\mathcal{S}\cdot\tilde{\alpha}_{-n}}\,|k,m,n=0\rangle\,. \end{aligned} \tag{4.27}$$

With this information and using the explicit form of the crosscaps given in appendix C, we can compute the required amplitudes. We obtain the corresponding amplitudes in the open channel with the usual modular transformations. After these operations we find in the open channel:

$$\begin{aligned} \mathcal{A} = {} & \frac{1}{2}(m_r)^2\,V_{r+1}\,(8\pi^2\alpha')^{-\left(\frac{r+1}{2}\right)} \int_0^\infty \frac{dt}{2t}\,t^{-\left(\frac{r+1}{2}\right)} \left\{\left(\frac{f_3(\tilde{q})}{f_1(\tilde{q})}\right)^8 - \left(\frac{f_2(\tilde{q})}{f_1(\tilde{q})}\right)^8\right\} \times \\ & \times \left\{\left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-2\pi t\frac{(mR)^2}{\alpha'}}\right)^4 + \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-2\pi t\frac{R^2}{\alpha'}(m+\frac{y}{\pi R})^2}\right)^4\right\}, \end{aligned} \tag{4.28}$$

with $\tilde{q} = \mathrm{e}^{-\pi t}$.

$$\begin{aligned} \mathcal{M}_9 = {} & -m_r\,V_{r+1}\,2^{\frac{7-r}{2}}\,(8\pi^2\alpha')^{-\left(\frac{r+1}{2}\right)} \int_0^\infty \frac{dt}{2t}\,t^{-\left(\frac{r+1}{2}\right)} \times \\ & \times \left\{\mathrm{e}^{i\frac{\pi}{4}(r-5)}\left(\frac{f_4(i\tilde{q})}{f_1(i\tilde{q})}\right)^{r-1}\left(\frac{f_3(i\tilde{q})}{f_2(i\tilde{q})}\right)^{9-r} - \right. \\ & \left. - \mathrm{e}^{-i\frac{\pi}{4}(r-5)}\left(\frac{f_3(i\tilde{q})}{f_1(i\tilde{q})}\right)^{r-1}\left(\frac{f_4(i\tilde{q})}{f_2(i\tilde{q})}\right)^{9-r}\right\}, \end{aligned} \tag{4.29}$$

and

$$\begin{aligned} \mathcal{M}_5 = {} & m_r\,V_{r+1}\,2^{\frac{3-r-s}{2}}\,(8\pi^2\alpha')^{-\left(\frac{r+1}{2}\right)} \int_0^\infty \frac{dt}{2t}\,t^{-\left(\frac{r+1}{2}\right)} \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-8\pi t\frac{R^2}{\alpha'}(m+\frac{y}{2\pi R})^2}\right)^4 \times \\ & \times \left\{\mathrm{e}^{-i\frac{\pi}{4}(1-r)}\left(\frac{f_4(i\tilde{q})}{f_1(i\tilde{q})}\right)^{3+r}\left(\frac{f_3(i\tilde{q})}{f_2(i\tilde{q})}\right)^{5-r} - \mathrm{e}^{i\frac{\pi}{4}(1-r)}\left(\frac{f_3(i\tilde{q})}{f_1(i\tilde{q})}\right)^{3+r}\left(\frac{f_4(i\tilde{q})}{f_2(i\tilde{q})}\right)^{5-r}\right\}. \end{aligned}$$

As was the case for the D-particle, in the previous section, to test the cancellation of the tachyons we make an expansion in powers of $\tilde{q}$, for $t \to \infty$ and we obtain the following leading term in the amplitude $\mathcal{A}_{\text{total}} = \mathcal{A} + \mathcal{M}_9 + \mathcal{M}_5$:

$$\begin{aligned} \mathcal{A}_{\text{total}} \simeq {} & \frac{m_r}{2}\,V_{r+1}\,(8\pi^2\alpha')^{-\left(\frac{r+1}{2}\right)} \int \frac{dt}{2t} t^{-\left(\frac{r+1}{2}\right)}\,\tilde{q}^{-1} \times \\ & \times \left\{\left(m_r - 2\sin\left[\frac{\pi}{4}(r-5)\right]\right) + \left(m_r - 2\sin\left[\frac{\pi}{4}(1-r)\right]\right)\mathrm{e}^{-8t\frac{y^2}{\alpha'\pi}}\right\}, \end{aligned} \tag{4.30}$$

which corresponds to the contribution from the tachyon. In order that the tachyons are projected out from the spectrum this contribution must vanish, hence the following condition must be satisfied:

$$m_r = 2\sin\left[\frac{\pi}{4}(1-r)\right]. \tag{4.31}$$

Note that if we take $r = 0$, which corresponds to the D-particle, we get $m_0 = \sqrt{2}$, which is the normalisation factor we have found for the tension of the D-particle using open-closed consistency condition. Besides the D-particle, we find that the only other stable non-BPS brane is the $(-1, 0)$-brane, i.e. a D-instanton for which we have $m_{-1} = 2$. For all other possible values of r $(= 1, 2, 4, 5)$ m_r is either zero or negative, hence they are not allowed. It is worth noting that this D-instanton, having the mass twice that of the BPS D-instanton of the type-II theory but with zero RR charge, is possibly just the superposition of the instanton and the anti-instanton. In the next section we will find another instanton but carrying a twisted RR charge. It may well be two different descriptions of the same instanton just like the familiar stable D-particle in type IIB in the orbifold $\mathbb{T}^4/(I_4(-1)^{F_L})$.

We now analyse the tachyons in the strings stretching to the tadpole-cancelling branes. For convenience, we consider the case for general (r, s) branes, keeping in mind that for $\mathbb{Z}_2$ branes only the case $s = 0$ is applicable. For the strings stretching between the non-BPS brane and the D9-brane we have

$$M^2 = \sum_{s\,(NN)\,\text{directions}} \left(\frac{n_i}{R_i}\right)^2 + \frac{1}{\alpha'}\left(\frac{5-p}{8}\right). \tag{4.32}$$

For $p = r + s \leq 5$ there are no tachyons, for any radius. Since the ground state of this sector ($n_i = 0$, $\forall i$) is not projected out of the spectrum, when $p > 5$ the brane contains always at least one tachyon. On the other hand, for the strings stretching between the non-BPS brane and the D5-brane we have

$$M^2 = \sum_{4-s\,(DD)\,\text{directions}} \left(\frac{m_i R_i}{\alpha'}\right)^2 + \frac{1}{\alpha'}\left(\frac{1-r+s}{8}\right). \tag{4.33}$$

For $r - s \leq 1$ there are no tachyons, for any radius. Similarly, for $r - s > 1$ the brane contains at least one tachyon in this sector. For $\mathbb{Z}_2$ branes this implies that there are no tachyons in any of these sectors if $r \leq 1$, hence the D-particle $(0, 0)$ and the D-instanton $(-1, 0)$ are absolutely stable non-BPS branes with $\mathbb{Z}_2$ charge.

4.3 Stable non-BPS branes with $\mathbb{Z}$ charge

These are the branes that carry a twisted R-R sector, namely

$$r = -1, 3\,, \qquad s = 0, 1, 3, 4\,, \tag{4.34}$$

and they have the following form:

$$|D(r,s)\rangle_{\mathbb{T}^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_{\rm NS,U} + |D(r,s)\rangle_{\rm R,T} + |C5\rangle + |C9\rangle\right). \tag{4.35}$$

Note that we do not take the combination of brane and its image, rather we use the formal definition and we localize the branes at the origin. As for the truncated branes in a type-IIB orbifold, they may be stable for some range of the radii. As before, we can obtain the amplitude of open strings on one of these branes using the boundary states:

$$\begin{aligned}\mathcal{A}_{\rm total} &= {}_{\mathbb{T}^4/\mathbb{Z}_2}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\mathbb{T}^4/\mathbb{Z}_2}\\ &= \mathcal{A}_{\text{NS-NS}} + \mathcal{A}_{\text{R-R},T} + \mathcal{M}_{9,\text{NS-NS}} + \mathcal{M}_{5,\text{NS-NS}}\,.\end{aligned} \tag{4.36}$$

The correspondence between the spin structures of the loop and tree channels is as follows:

$$\begin{aligned}\mathcal{A}_{\text{NS-NS}} &= \int_0^\infty \frac{dt}{2t}\,{\rm Tr}_{{\rm NS},R}\left(\frac{1}{4}(-1)^{F_s}{\rm e}^{-2\pi t L_0}\right) = \frac{1}{2}{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\rm NS,U}\,,\\
\mathcal{A}_{\text{R-R},T} &= \int_0^\infty \frac{dt}{2t}\,{\rm Tr}_{\rm NS}\left(\frac{1}{4}(-1)^{F+G}\mathcal{I}_4\,{\rm e}^{-2\pi t L_0}\right) = \frac{1}{2}{}_{\rm R,T}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\rm R,T}\,,\\
\mathcal{M}_{9,\text{NS-NS}} &= \int_0^\infty \frac{dt}{2t}\,{\rm Tr}_{\rm NS}\left(\frac{1}{4}\Omega\,{\rm e}^{-2\pi t L_0}\right) = \frac{1}{2}{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|C9\rangle_{\rm NS,U} + {\rm c.c.}\,,\\
\mathcal{M}_{5,\text{NS-NS}} &= \int_0^\infty \frac{dt}{2t}\,{\rm Tr}_{\rm NS}\left(\frac{1}{4}(-1)^{F+G}\mathcal{I}_4\Omega{\rm e}^{-2\pi t L_0}\right)\\
&= \frac{1}{2}{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|C5\rangle_{\rm NS,U} + {\rm c.c.}\,,\end{aligned}$$

where F_s is the space-time fermion number and $(-1)^{F+G}$ is the GSO projection operator including the super-ghosts. Furthermore, we take into account that

$${\rm Tr}_R\left((-1)^{F+G}\mathcal{I}_4\,{\rm e}^{-2\pi t L_0}\right) = 0\,, \tag{4.37}$$

due to the presence of zero modes in the R-vacuum, and

$${\rm Tr}_R\left(\Omega\,{\rm e}^{-2\pi t L_0}\right) = 0\,, \tag{4.38}$$

since Ω acts as a product of gamma-matrices on the R-vacuum, and finally

$${\rm Tr}_R\left((-1)^{F+G}\mathcal{I}_4\Omega\,{\rm e}^{-2\pi t L_0}\right) = 0\,, \tag{4.39}$$

since although the action of Ω and $\mathcal{I}_4$ compensate it still vanishes due to $(-1)^{F+G}$. Accordingly we can write the total amplitude in terms of an open string loop as follows:

$$\mathcal{A}_{\text{total}} = \int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}\left\{(-1)^{F_s}\left(\frac{1+(-1)^{F+G}\mathcal{I}_4}{2}\right)\left(\frac{1+\Omega}{2}\right)\mathrm{e}^{-2\pi t L_0}\right\}. \tag{4.40}$$

The open strings on these D-branes have winding and Kaluza-Klein modes

$$M^2 = \sum_{4-s(DD)\,\text{directions}}\left(\frac{mR}{\alpha'}\right)^2 + \sum_{s(NN)\,\text{directions}}\left(\frac{n}{R}\right)^2 + \frac{1}{\alpha'}\left(N-\frac{1}{2}\right), \tag{4.41}$$

hence for a certain range of the radii there are no tachyons in the spectrum: $R \geq \sqrt{\alpha'/2}$ for the DD-directions and $R \leq \sqrt{2\alpha'}$ for the NN-directions. Moreover, at particular radii, the tachyon modes become massless. In the case of non-BPS branes in type-II orbifolds, these critical radii coincide with the critical radii where the brane has a vanishing 1-loop amplitude [28]. In this case, however, this is not possible since there is a remaining NS-NS interaction of the brane with the crosscaps that cannot be cancelled.

In order to carry out the analysis of tachyons in the strings stretched from the non-BPS brane to the tadpole-cancelling branes we can use the analysis carried out before for the $\mathbb{Z}_2$-charged non-BPS branes. Using (4.32) and (4.33) one can see that the *instantonic* branes $|D(-1,0)\rangle_{\mathbb{T}^4/Z_2}$, $|D(-1,1)\rangle_{\mathbb{T}^4/Z_2}$, $|D(-1,3)\rangle_{\mathbb{T}^4/Z_2}$ and $|D(-1,4)\rangle_{\mathbb{T}^4/Z_2}$ have no tachyons in this sector at any radius.

On the other hand, the branes $|D(3,0)\rangle_{\mathbb{T}^4/Z_2}$, $|D(3,1)\rangle_{\mathbb{T}^4/Z_2}$, $|D(3,3)\rangle_{\mathbb{T}^4/Z_2}$ and $|D(3,4)\rangle_{\mathbb{T}^4/Z_2}$ contain at least a tachyon in this sector. Accordingly, the *instantonic* truncated branes listed above are fully stable if the radii take the values $R \geq \sqrt{\alpha'/2}$ for the DD-directions and $R \leq \sqrt{2\alpha'}$ for the NN-directions, for each particular brane. Finally, we observe that the D-instanton of type I, which becomes a truncated brane with twisted R-R charge in the type-I orbifold, can be stable in such theory.

5. Conclusions

In this article we have presented a thorough analysis of the action of the Ω projection on the boundary states of type-IIB theory in a $\mathbb{T}^4/\mathcal{I}_4$ orbifold. Of particular interest are the R-R sectors and the twisted NS-NS sector. We have shown how to derive the action of Ω in the R-R sectors of the covariant boundary states, which are formulated in the asymmetric picture $(-1/2,-3/2)$, for which the superghost zero-modes play an important role. In the twisted NS-NS sector, where there are no superghost zero modes, we have implemented the action of Ω through the algebra of fermionic zero modes. Since these results rely on basic features of the covariant boundary states, which are common to other theories, our approach could be extended to other orbifold and orientifold theories in order to derive their complete D-brane spectrum in a systematic way.

Using the results of our analysis of the Ω projection, and taking into account the action of the orbifold on the different sectors, we have derived which boundary states are present in the type-I orbifold. From this we have derived the spectrum of BPS and non-BPS D-branes of type I on $\mathbb{T}^4/\mathcal{I}_4$. Regarding the non-BPS D-branes, they are divided into $\mathbb{Z}$-charged branes, which have a twisted R-R sector, and $\mathbb{Z}_2$-charged branes, which are described by a untwisted NS-NS sector only.

The analysis of the stability of these non-BPS D-branes has been carried out. For the $\mathbb{Z}_2$-charged non-BPS branes we have found that only branes of the type $(r, 0)$ can be formulated as such. From those, only the particle $|D(0,0)\rangle$ and the instanton $|D(-1,0)\rangle$ are absolutely stable since they do not contain any tachyon in their spectrum of open strings.

On the other hand, the non-BPS $\mathbb{Z}$-charged branes have no tachyons on their brane spectrum for a particular range of the radii, namely, $R \geq \sqrt{\alpha'/2}$ for the DD-directions and $R \leq \sqrt{2\alpha'}$ for the NN-directions, as occurs in the type-IIB orbifold. We have found that, moreover, only the branes $|D(-1,0)\rangle$, $|D(-1,1)\rangle$, $|D(-1,3)\rangle$, $|D(-1,4)\rangle$, do not have any tachyons in the open strings stretching to the tadpole cancelling branes, so they can be considered as fully stable for the range of the radii mentioned above.

It is interesting to analyse the fate of the well-known non-BPS branes of type-I theory after the orbifolding. The non-BPS D-particle becomes a stable non-BPS $\mathbb{Z}_2$-charged brane. D7- and D8-branes cannot be formulated as $\mathbb{Z}_2$-charged branes, and the D8-brane cannot carry any R-R charge, hence there are no D8-branes in the type-I orbifold. There is, however, a non-BPS D7-branes with twisted R-R charge for the orientation $(3, 4)$, but it is unstable since it has tachyons in the open strings stretching to the tadpole-cancelling branes. Finally, the non-BPS D-instanton of type I becomes a stable non-BPS brane and appears as a $\mathbb{Z}$-charged brane when it sits at any of the fixed planes and all radii fulfill $R \geq \sqrt{\alpha'/2}$, and as a $\mathbb{Z}_2$-charged brane when it is separated from the fixed planes.

It would be of great interest to extend the techniques used in this article to other orientifolds where the non-BPS branes play a more relevant role in the consistency of the theory [17, 18, 29] and in particular to those models with a phenomenological interest [30].

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A. Ω and the asymmetric picture

In this appendix we give a detailed computation of the action of Ω on the boundary states formulated in the asymmetric superghost picture $(-1/2, -3/2)$. The action of Ω on the (untwisted) R-R boundary states in the light-cone gauge formulation was derived before in [22]. However, only the covariant formulation [21] allows the description of D-branes which are of type spacetime-filling or domain-wall with respect to either the bulk or the orbifold fixed planes, hence the relevance of understanding the action of Ω in this formulation. For instance, in the present case, it allows the description of the untwisted R-R sector of a D9-brane and the twisted R-R sector of any D-brane for which all the spatial directions of the orbifold fixed plane are Neumann-directions.

Usually, in order to find out which (BPS) D-branes survive the Ω projection, one may consider the R-R state in the $(-1/2, -1/2)$ picture:

$$|F_{p+2}\rangle = \frac{1}{(p+2)!} F_{\mu_0\ldots\mu_{p+1}} \left(\mathcal{C}_{(10)}\Gamma^{\mu_0\ldots\mu_{p+1}}\right)_{AB} |A\rangle_{-1/2} \otimes \widetilde{|B\rangle}_{-1/2} . \qquad (A.1)$$

Applying Ω and taking into account that for the R-R vacuum

$$\Omega\left(|A\rangle_{-1/2} \otimes \widetilde{|B\rangle}_{-1/2}\right) = \widetilde{|A\rangle}_{-1/2} \otimes |B\rangle_{-1/2} = -|B\rangle_{-1/2} \otimes \widetilde{|A\rangle}_{-1/2} , \qquad (A.2)$$

we find:

$$\Omega\,|F_{p+2}\rangle = -(-1)^{\frac{1}{2}(p+1)(p+2)}\,|F_{p+2}\rangle , \qquad (A.3)$$

where we have used that $p = odd$ for type IIB and that $\mathcal{C}^T_{(10)} = -\mathcal{C}_{(10)}$. This state is left invariant for $p = 1, 5$, and changes sign for $p = -1, 3, 7$. Notice that here it is crucial that the picture is left-right symmetric, in order to recover the original ordering of the pictures.

On the other hand, the boundary state of the R-R sector in the covariant formulation is constructed upon the vacuum in the asymmetric picture $(-1/2, -3/2)$ [25]. This boundary state is given by[15]

$$|D\rangle_{\rm R,U} = \frac{1}{2}\left(|D,+\rangle_{\rm R,U} + |D,-\rangle_{\rm R,U}\right) , \qquad (A.4)$$

with

$$|D,\eta\rangle_{\rm R,U} = \frac{Q}{2}\,|D_X\rangle|D_\psi,\eta\rangle_{\rm R,U}|D_{\rm gh}\rangle|D_{\rm sgh},\eta\rangle_{\rm R} , \qquad (A.5)$$

and

$$\begin{aligned} |D_{\rm sgh},\eta\rangle^{(0)}_{\rm R}|D_\psi,\eta\rangle^{(0)}_{\rm R,U} &= {\rm e}^{i\eta\gamma_0\tilde\beta_0}\mathcal{M}_{AB}\,|A\rangle_{-1/2} \otimes \widetilde{|B\rangle}_{-3/2} , \\ \mathcal{M}_{AB}(\eta) &= \left(\mathcal{C}_{(10)}\Gamma^0\cdots\Gamma^p\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB} , \qquad p = r+s . \end{aligned} \qquad (A.6)$$

[15]For simplicity, we omit the label (r,s) in what follows.

However, under the action of Ω, the superghost pictures are interchanged:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right)=\widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-3/2}=-|B\rangle_{-3/2}\otimes\widetilde{|A\rangle}_{-1/2}\,. \tag{A.7}$$

In this case, in order to bring it back to the $(-1/2,-3/2)$ picture, a more careful treatment is needed. In order to understand how Ω acts in the states with asymmetric picture we introduce the formal operators [31]

$$\delta(\gamma_m)\,,\qquad \delta(\beta_n)\,, \tag{A.8}$$

for each of the holomorphic sectors, which allow to relate different pictures

$$\delta(\beta_{-q-3/2})|q\rangle=|q+1\rangle\,,\qquad \delta(\gamma_{q+1/2})|q\rangle=|q-1\rangle\,. \tag{A.9}$$

Moreover, they have the following commutation relations:

$$\begin{aligned}
&[\beta_m,\delta(\beta_n)]=0\,, &&[\gamma_m,\delta(\beta_n)]=\delta_{n,-m}\frac{d}{d\beta_n}\delta(\beta_n)\,,\\
&[\gamma_m,\delta(\gamma_n)]=0\,, &&[\beta_m,\delta(\gamma_n)]=-\delta_{n,-m}\frac{d}{d\gamma_n}\delta(\gamma_n)\,.
\end{aligned}$$

Therefore, the two possible vacuum states of the superghost zero-modes

$$\beta_0|{\downarrow}\rangle=0\,,\qquad \gamma_0|{\uparrow}\rangle=0\,, \tag{A.10}$$

can be characterised as follows:

$$\delta(\beta_0)|{\uparrow}\rangle=|{\downarrow}\rangle\,,\qquad \delta(\gamma_0)|{\downarrow}\rangle=|{\uparrow}\rangle\,, \tag{A.11}$$

in analogy with the two vacuum states of the ghost zero-modes. On the other hand, considering the overlapping conditions for the zero-mode of the superghost boundary state

$$(\gamma_0+i\eta\tilde{\gamma}_0)|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)}=0\,,\qquad (\beta_0+i\eta\tilde{\beta}_0)|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)}=0\,, \tag{A.12}$$

we can find the following solution to these conditions

$$|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)}=\delta(\gamma_0+i\eta\tilde{\gamma}_0)\,|{\downarrow}\rangle\otimes\widetilde{|{\downarrow}\rangle}\,. \tag{A.13}$$

Using the above results, we can rewrite the zero-mode part of the superghost boundary state in the asymmetric $(-1/2,-3/2)$ picture as follows:

$$\begin{aligned}
|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)}&={\rm e}^{i\eta\gamma_0\tilde{\beta}_0}\,|{-1/2}\rangle\otimes\widetilde{|{-3/2}\rangle}\\
&={\rm e}^{i\eta\gamma_0\tilde{\beta}_0}\,\delta(\tilde{\gamma}_0)\,|{-1/2}\rangle\otimes\widetilde{|{-1/2}\rangle}\\
&=\delta(\tilde{\gamma}_0-i\eta\,\gamma_0)\,|{-1/2}\rangle\otimes\widetilde{|{-1/2}\rangle}\,,
\end{aligned} \tag{A.14}$$

which allows to easily derive its transformation rule under Ω:

$$\begin{aligned}\Omega|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)} &= \delta(\gamma_0 - i\eta\tilde{\gamma}_0)\,\widetilde{|-1/2\rangle}\otimes|-1/2\rangle \\ &= -i\eta\,\delta(\tilde{\gamma}_0 + i\eta\gamma_0)\,|-1/2\rangle\otimes\widetilde{|-1/2\rangle} \\ &= -i\eta\,|D_{\rm sgh},-\eta\rangle_{\rm R}^{(0)}\,.\end{aligned} \tag{A.15}$$

Furthermore, for the complete superghost state

$$|D_{\rm sgh},\eta\rangle_{\rm R} = \exp\left(i\eta\sum_{n=1}^{\infty}(\gamma_{-n}\tilde{\beta}_{-n} - \beta_{-n}\tilde{\gamma}_{-n})\right)|D_{\rm sgh},\eta\rangle_{\rm R}^{(0)}\,, \tag{A.16}$$

using that Ω exchange left and right superghost oscillators, we finally obtain:

$$\Omega|D_{\rm sgh},\eta\rangle_{\rm R} = -i\eta|D_{\rm sgh},-\eta\rangle_{\rm R}\,. \tag{A.17}$$

Using the above result we can also derive the Ω-projection on the D-branes by using the form of the R-R string state in the $(-1/2,-3/2)$ picture:

$$\begin{aligned}|C^{(p+1)}\rangle = \frac{1}{2(p+1)!\sqrt{2}}\,C^{(p+1)}_{\mu_1\cdots\mu_{p+1}}&\Big(\big(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_+\big)_{AB}\cos\gamma_0\tilde{\beta}_0 + \\ &+\big(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_-\big)_{AB}\sin\gamma_0\tilde{\beta}_0\Big)\times \\ \times\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,.\end{aligned} \tag{A.18}$$

This analysis is very useful for checking whether there are spacetime-filling D-branes in the spectrum. We have derived above that for the zero modes:

$$\Omega\left({\rm e}^{i\eta\gamma_0\tilde{\beta}_0}|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) = i\eta\,{\rm e}^{-i\eta\gamma_0\tilde{\beta}_0}\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,, \tag{A.19}$$

which implies

$$\begin{aligned}\Omega\left(\cos\gamma_0\tilde{\beta}_0\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) &= \sin\gamma_0\tilde{\beta}_0\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,, \\ \Omega\left(\sin\gamma_0\tilde{\beta}_0\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) &= \cos\gamma_0\tilde{\beta}_0\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,.\end{aligned} \tag{A.20}$$

Finally, using this result and the properties of the Γ-matrices we find:

$$\Omega\,|C^{(p+1)}\rangle = -(-1)^{\frac{1}{2}p(p+1)}\,|C^{(p+1)}\rangle\,, \tag{A.21}$$

hence it is invariant for $p = 1, 5, 9$, as seen before.

Likewise, we can find the action of Ω on the quantum state of the R-R twisted sector in the $(-1/2,-3/2)$ picture:

$$\begin{aligned}|C^{(r+1)}\rangle_{\rm T} = \frac{1}{\sqrt{2}(p+1)!}C^{(r+1)}_{\mu_1\cdots\mu_{r+1}}&\Big(\big(\mathcal{C}_{(6)}\gamma^{\mu_1\cdots\mu_{p+1}}\Pi_+\big)_{ab}\cos\gamma_0\tilde{\beta}_0 \\ &+\big(\mathcal{C}_{(6)}\gamma^{\mu_1\cdots\mu_{p+1}}\Pi_-\big)_{ab}\sin\gamma_0\tilde{\beta}_0\Big)\times \\ \times\,|a\rangle_{-1/2,{\rm T}}\otimes\widetilde{|b\rangle}_{-3/2,{\rm T}}\,.\end{aligned} \tag{A.22}$$

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Using (A.20) and the properties of the 6-dimensional γ-matrices we find:

$$\Omega\,|C^{(r+1)}\rangle_{\rm T} = (-1)^{\frac{1}{2}r(r+1)}\,|C^{(r+1)}\rangle_{\rm T}\,, \tag{A.23}$$

hence it is invariant for $r=-1$, and 3.

B. Action of Ω on the twisted NS-NS vacuum

In this appendix we give an explicit analysis of the action of Ω in the twisted NS-NS vacuum. This vacuum state consists of the spin fields constructed out of the four fermionic zero modes in the left and four from the right moving sector. The superghost vacuum is taken to be in the $(-1,-1)$ picture. In each of the left and right sector, these fermion zero modes satisfy an SO(4) Clifford algebra. Thus we have

$$\{\psi_0^i,\psi_0^j\} = \delta^{ij} = \{\tilde\psi_0^i,\tilde\psi_0^j\}\,, \qquad \{\psi_0^i,\tilde\psi_0^j\} = 0\,. \tag{B.1}$$

where i,j take values from 6,7,8 and 9. Our conventions for the SO(4) gamma-matrices are as follows:

$$\{\bar\gamma^i,\bar\gamma^j\} = 2\delta^{ij}\,, \qquad \mathcal{C}_{(4)}^T = -\mathcal{C}_{(4)}\,, \qquad [\mathcal{C}_{(4)},\bar\gamma] = 0\,, \qquad (\bar\gamma^i)^T = -\mathcal{C}_{(4)}\bar\gamma^i\mathcal{C}_{(4)}^{-1}\,, \tag{B.2}$$

with $\bar\gamma = -\bar\gamma^6\bar\gamma^7\bar\gamma^8\bar\gamma^9$. Let α,β denote the spinor indices in four dimensions and let $|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T}$ denote the twisted spinor vacuum constructed from the spin fields of the above fermionic zero modes. The action of the fermionic oscillators and zero modes in this basis is defined to be

$$\begin{aligned}
\psi_n^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T} &= \tilde\psi_n^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T} = 0\,, \qquad \forall n\geq 1\,,\\
\psi_0^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T} &= \frac{1}{\sqrt2}\left(\bar\gamma^i\right)^\alpha{}_\delta\left(\mathbb{1}\right)^\beta{}_\rho\,|\delta\rangle_{\rm T}\otimes\widetilde{|\rho\rangle}_{\rm T}\,,\\
\tilde\psi_0^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T} &= \frac{1}{\sqrt2}\left(\bar\gamma\right)^\alpha{}_\delta\left(\bar\gamma^i\right)^\beta{}_\rho\,|\delta\rangle_{\rm T}\otimes\widetilde{|\rho\rangle}_{\rm T}\,.
\end{aligned} \tag{B.3}$$

As in the earlier cases, the action of Ω is to interchange the left and right moving sectors. Thus the defining relation for the action of Ω can be obtained by demanding simply that, namely

$$\Omega\left(\tilde\psi_0^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T}\right) = \psi_0^i\widetilde{|\alpha\rangle}_{\rm T}\otimes|\beta\rangle_{\rm T}\,, \tag{B.4}$$

or equivalently

$$\Omega\left(\psi_0^i|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T}\right) = \tilde\psi_0^i\widetilde{|\alpha\rangle}_{\rm T}\otimes|\beta\rangle_{\rm T}\,, \tag{B.5}$$

where

$$\widetilde{|\alpha\rangle}_{\rm T}\otimes|\beta\rangle_{\rm T} = -|\beta\rangle_{\rm T}\otimes\widetilde{|\alpha\rangle}_{\rm T}\,. \tag{B.6}$$

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These definitions guarantee that $\Omega^2 = 1$. Using the equations in (B.3), the first equation (B.4) gives:

$$\left(\bar{\gamma}\right)^{\alpha}{}_{\delta}\left(\bar{\gamma}^{i}\right)^{\beta}{}_{\rho}\,\Omega\left(|\delta\rangle_{\rm T}\otimes\widetilde{|\rho\rangle}_{\rm T}\right) = -\left(\bar{\gamma}^{i}\right)^{\beta}{}_{\rho}\,(\mathbb{1})^{\alpha}{}_{\delta}\,|\rho\rangle_{\rm T}\otimes\widetilde{|\delta\rangle}_{\rm T}\,, \tag{B.7}$$

which can be simplified to obtain

$$\Omega\left(|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T}\right) = -\left(\bar{\gamma}\right)^{\alpha}{}_{\delta}\,|\beta\rangle_{\rm T}\otimes\widetilde{|\delta\rangle}_{\rm T}\,. \tag{B.8}$$

Similarly, the equation (B.5) gives rise to

$$\left(\bar{\gamma}^{i}\right)^{\alpha}{}_{\delta}\,(\mathbb{1})^{\beta}{}_{\rho}\,\Omega\left(|\delta\rangle_{\rm T}\otimes\widetilde{|\rho\rangle}_{\rm T}\right) = -\left(\bar{\gamma}\right)^{\beta}{}_{\rho}\left(\bar{\gamma}^{i}\right)^{\alpha}{}_{\delta}\,|\rho\rangle_{\rm T}\otimes\widetilde{|\delta\rangle}_{\rm T}\,, \tag{B.9}$$

which can be simplified to obtain

$$\Omega\left(|\alpha\rangle_{\rm T}\otimes\widetilde{|\beta\rangle}_{\rm T}\right) = -\left(\bar{\gamma}\right)^{\beta}{}_{\delta}\,|\delta\rangle_{\rm T}\otimes\widetilde{|\alpha\rangle}_{\rm T}\,. \tag{B.10}$$

If we denote the superghost vacuum in the $(-1,-1)$ picture as $|-1\rangle\otimes\widetilde{|-1\rangle}$, with

$$\Omega\left(|-1\rangle\otimes\widetilde{|-1\rangle}\right) = \widetilde{|-1\rangle}\otimes|-1\rangle = -|-1\rangle\otimes\widetilde{|-1\rangle}\,, \tag{B.11}$$

and noting that the full vacuum is given by

$$|\alpha\rangle_{-1,\rm T}\otimes\widetilde{|\beta\rangle}_{-1,\rm T} = |\alpha\rangle_{\rm T}|-1\rangle\otimes\widetilde{|\beta\rangle}_{\rm T}\widetilde{|-1\rangle}\,, \tag{B.12}$$

we can write the action of Ω on the full vacuum of the twisted NS-NS sector as follows:

$$\Omega\left(|\alpha\rangle_{-1,\rm T}\otimes\widetilde{|\beta\rangle}_{-1,\rm T}\right) = (\bar{\gamma})^{\alpha}{}_{\delta}\,|\beta\rangle_{-1,\rm T}\otimes\widetilde{|\delta\rangle}_{-1,\rm T} = (\bar{\gamma})^{\beta}{}_{\delta}\,|\delta\rangle_{-1,\rm T}\otimes\widetilde{|\alpha\rangle}_{-1,\rm T}\,, \tag{B.13}$$

as previously announced in (2.34). Finally, note that this relation is equivalent to

$$|\alpha\rangle_{-1,\rm T}\otimes\widetilde{|\beta\rangle}_{-1,\rm T} = (\bar{\gamma})^{\alpha}{}_{\delta}\,(\bar{\gamma})^{\beta}{}_{\rho}\,|\delta\rangle_{-1,\rm T}\otimes\widetilde{|\rho\rangle}_{-1,\rm T}\,, \tag{B.14}$$

which is consistent since both left and right spinors are of the same chirality.

C. Crosscaps in type I on $\mathbb{T}^4/\mathbb{Z}_2$

The tadpole-cancelling D9- and D5-branes in type I on $\mathbb{T}^4/\mathcal{I}_4$ are bulk branes. In type I on a K3 orbifold we have a 9-crosscap and 5-crosscaps which have boundary states in the untwisted sectors only:

$$|C9\rangle = |C9\rangle_{\rm NS} + |C9\rangle_{\rm R}\,, \qquad |C5\rangle = |C5\rangle_{\rm NS} + |C5\rangle_{\rm R}\,. \tag{C.1}$$

As usual, each part is defined in terms of the spin structures:

$$\begin{aligned} |C\rangle_{\rm NS} &= \frac{1}{2}\left(|C,+\rangle_{\rm NS} - |C,-\rangle_{\rm NS}\right),\\ |C\rangle_{\rm R} &= \frac{1}{2}\left(|C,+\rangle_{\rm R} + |C,-\rangle_{\rm R}\right), \end{aligned} \tag{C.2}$$

where

$$|C,\eta\rangle_{\rm NS/R} = |C_X\rangle|C_\psi,\eta\rangle_{\rm NS/R}|C_{\rm gh}\rangle|C_{\rm sgh},\eta\rangle_{\rm NS/R}\,. \tag{C.3}$$

This generic form is common to both crosscaps:

$$
\begin{aligned}
|C_\psi, \eta\rangle_{\rm NS} &= \prod_{r=1/2}^{\infty} \mathrm{e}^{i\eta(-1)^r \psi_{-r}\cdot S\cdot\tilde{\psi}_{-r}}\, |0\rangle\,, \\
|C_{\rm sgh}, \eta\rangle_{\rm NS} &= \prod_{r=1/2}^{\infty} \mathrm{e}^{i\eta(-1)^r (\gamma_{-r}\tilde{\beta}_{-r} - \beta_{-r}\tilde{\gamma}_{-r})}\, |-1\rangle \otimes \widetilde{|-1\rangle}\,, \\
|C_{\rm gh}\rangle &= \prod_{n=1}^{\infty} \mathrm{e}^{(-1)^n (c_{-n}\tilde{b}_{-n} - b_{-n}\tilde{c}_{-n})} \left(\frac{c_0+\tilde{c}_0}{2}\right) |1\rangle \otimes \widetilde{|1\rangle}\,, \\
|C_\psi, \eta\rangle_{\rm R} &= \prod_{m=1}^{\infty} \mathrm{e}^{i\eta(-1)^m \psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}}\, |C_\psi, \eta\rangle_{\rm R}^{(0)}\,, \\
|C_{\rm sgh}, \eta\rangle_{\rm R} &= \prod_{m=1}^{\infty} \mathrm{e}^{i\eta(-1)^m (\gamma_{-m}\tilde{\beta}_{-m} - \beta_{-m}\tilde{\gamma}_{-m})}\, |C_{\rm sgh}, \eta\rangle_{\rm R}^{(0)}\,,
\end{aligned}
\tag{C.4}
$$

with $S_{\mu\nu} = \eta_{\mu\nu}$ for the 9-crosscap and $S_{\mu\nu} = (\eta_{\alpha\beta}, -\delta_{ij})$, for the 5-crosscap, where $\alpha, \beta = 0, \ldots, 5$ are the directions on the fixed planes and $i, j = 6, \ldots, 9$ are the directions along $\mathbb{T}^4$. The zero modes in the R-R sector are given by:

$$
\begin{aligned}
|C_\psi, \eta\rangle_{\rm R}^{(0)} &= \left(\mathcal{C}_{(10)}\Gamma^0 \cdots \Gamma^p \frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB} |A\rangle \otimes \widetilde{|B\rangle}\,, \qquad p = 5, 9\,, \\
|C_{\rm sgh}, \eta\rangle_{\rm R}^{(0)} &= \mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}\, |-1/2\rangle \otimes \widetilde{|-3/2\rangle}\,.
\end{aligned}
\tag{C.5}
$$

The fact that the orbifold is compact is reflected in the bosonic-oscillator part of the crosscaps states. The 9-crosscap contains windings along the $\mathbb{T}^4$:

$$
|C9_X\rangle = \mathcal{N}_9 \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{iq_m \frac{mR}{\alpha'}}\right)^4 \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n}(-1)^n \alpha_{-n}\cdot S\cdot\tilde{\alpha}_n}\, |0\rangle\,, \tag{C.6}
$$

whereas the 5-crosscap contains momentum modes along the $\mathbb{T}^4$ directions:

$$
|C5_X\rangle = \mathcal{N}_5 \left(\sum_{n\in\mathbb{Z}} \mathrm{e}^{iq_n \frac{n}{R}}\right)^4 \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n}(-1)^n \alpha_{-n}\cdot S\cdot\tilde{\alpha}_n}\, |0\rangle\,. \tag{C.7}
$$

For simplicity we have taken all four radii equal to each other. The normalisation of these crosscaps states is derived by comparing the open string Möbius amplitudes with a closed string calculation:

$$
\begin{aligned}
\mathcal{N}_9 &= \frac{1}{\sqrt{2}}\, 2^5\, \frac{T_9}{2} \left(\frac{2\pi R}{\Phi}\right)^2, \\
\mathcal{N}_5 &= -\sqrt{2}\, \frac{T_5}{2} \left(\frac{2\pi R}{\Phi}\right)^2 (2\pi R)^{-4}\,,
\end{aligned}
\tag{C.8}
$$

where the relative sign between $\mathcal{N}_9$ and $\mathcal{N}_5$ stems from the fact that Ω acts with a relative sign on the D9- and D5-branes [19] and T_p is given in (1.4).

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